Solving with Basic Identities (3.10, 3.11, 3.12) HW

Solve over the interval $0 \le \theta < 2\pi$. Be careful of extraneous solutions that are outside the domain of these functions.

1. $2\cos\theta\tan\theta = -\sqrt{2}$
2. $2\sin\theta + 5 = 3\csc\theta$

Solve over the interval $-\pi \le \theta < \pi$. Be careful of extraneous solutions that are outside the domain of these functions.

3. $\cos\theta - 2\sec\theta = 1$
4. $\frac{\tan\theta}{\csc\theta} = \sin\theta$

Find an expression or expressions that represent ALL solutions to each equation.

5. $\frac{\tan\theta}{\sec^2\theta - 1} = \sqrt{3}$
6. $(\csc\theta - 1)(\csc\theta + 1) = 3$
7. $2\cos\theta\cot\theta + \cot\theta = \csc\theta$
8. $\cos\theta - \tan\theta\sin\theta = 0$

In each question, four expressions are presented. Three of the expressions are equivalent where defined. One is not. Find the one expression that is not equivalent to the others.

9.	$\cos^4\theta - \sin^4\theta$	$\cos^2\theta - \sin^2\theta$	$2\sin^2\theta - 1$	$2\cos^2\theta - 1$
10.	$\dfrac{\cos\theta}{\tan\theta - \sec\theta}$	$\dfrac{(1+\sin\theta)(1-\sin\theta)}{\sin\theta}$	$\csc\theta - \sin\theta$	$\dfrac{\cos^2\theta}{\sin\theta}$
11.	$\sin^2\theta - \cos^2\theta$	$\tan^2\theta + \cot^2\theta + 2$	$\sec^2\theta + \csc^2\theta$	$(\tan\theta + \cot\theta)^2$
12.	$\dfrac{\tan\theta}{1+\tan^2\theta}$	$\cos^2\theta - \sin^2\theta$	$\dfrac{\cos\theta}{\csc\theta}$	$\sin\theta\cos\theta$

Solving Basic Identities HW Answers

1. $\theta = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
2. $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
3. $\theta = -\pi$
4. $\theta = -\frac{3\pi}{4}$ or $\frac{\pi}{4}$
5. $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{7\pi}{6} + 2\pi k$, where $k \in \mathbb{Z}$
   This may be written more concisely as $\theta = \frac{\pi}{6} + \pi k$, where $k \in \mathbb{Z}$
6. $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{5\pi}{6} + 2\pi k$ or $\theta = \frac{7\pi}{6} + 2\pi k$ or $\theta = \frac{11\pi}{6} + 2\pi k$, where $k \in \mathbb{Z}$
   This may be written more concisely as $\theta = \frac{\pi}{6} + \pi k$ or $\theta = \frac{5\pi}{6} + \pi k$, where $k \in \mathbb{Z}$
7. $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$
8. $\theta = \frac{\pi}{4} + 2\pi k$ or $\theta = \frac{3\pi}{4} + 2\pi k$ or $\theta = \frac{5\pi}{4} + 2\pi k$ or $\theta = \frac{7\pi}{4} + 2\pi k$, where $k \in \mathbb{Z}$
   This can be stated more concisely as $\theta = \frac{\pi}{4} + \frac{\pi}{2}k$, where $k \in \mathbb{Z}$
9. $2\sin^2\theta - 1$ is not equivalent.
$$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = (\cos^2\theta - \sin^2\theta)(1) = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$$
10. $\frac{\cos\theta}{\tan\theta - \sec\theta}$ is not equivalent.
$$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{(1+\sin\theta)(1-\sin\theta)}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$
11. $\sin^2\theta - \cos^2\theta$ is not equivalent.
$$(\tan\theta + \cot\theta)^2 = \tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = \tan^2\theta + 2\tan\theta\left(\frac{1}{\tan\theta}\right) + \cot^2\theta = \tan^2\theta + 2 + \cot^2\theta$$
$$= \tan^2\theta + 1 + \cot^2\theta + 1 = \sec^2\theta + \csc^2\theta$$
12. $\cos^2\theta - \sin^2\theta$ is not equivalent.
$$\frac{\tan\theta}{1+\tan^2\theta} = \frac{\tan\theta}{\sec^2\theta} = \frac{\sin\theta / \cos\theta}{1 / \cos^2\theta} = \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1} = \sin\theta\cos\theta = \frac{1}{\csc\theta}\cdot\cos\theta = \frac{\cos\theta}{\csc\theta}$$